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Study on Adjacency Matrix for Flow

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we introduced the adjacency matrix for flow problems and its cases, then we discussed how could we computed max flow by using this matrix.

1. Introduction

Adjacency matrix: In graph theory and computer science, an **adjacency matrix** is a square matrix used to represent a finite graph. The elements of the matrix indicate whether pairs of vertices are adjacent or not in the graph [1].

Max flow: In optimization theory, maximum flow problems involve finding a feasible flow through a single-source, single-sink flow network that is maximum [2,3].

2. Results and discussion

The adjacency matrix for flow problems take the shape:

$$\begin{bmatrix} A_{11} & A_{21} & A_{31} & \cdots & A_{n1} \\ A_{12} & A_{22} & A_{23} & \cdots & A_{n2} \\ A_{13} & A_{23} & A_{33} & \cdots & A_{n3} \\ A_{1n-1} & . & . & \cdots & . \\ . & . & . & & . \\ . & . & . & & \\ A_{1n} & A_{2n} & A_{3n} & \cdots & A_{nn} \end{bmatrix}$$

Where all values of this matrix take values as follow:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ A_{12} & 0 & 0 & 0 & 0 \\ A_{13} & A_{23} & 0 & 0 & 0 \\ 0 & A_{24} & . & 0 & 0 \\ . & . & . & . & \\ 0 & 0 & A_{3n} & A_{4n} & 0 \end{bmatrix}$$

Values of A_{1n-1} , A_{1n} , A_{2n} equals 0 .

- Each path of flow can be illustrated as the same matrix.

-We can compute max flow from this matrix by eliminating all other columns from the matrix except the columns of associated path, and then subtract the lowest flow from each weight of the associated matrix.

We can write these computations as algorithm as follows:

Algorithm:

Input: Adjacency matrix Mi of flow F.

1. Let P_1 be the first path from source (S) to sink (T) , M_1 is the adjacency matrix for P_1.
   a. Select the lowest flow X_1 on P_1.
   b. Subtract X_1 from all elements on M_1.
2. IF there exist P_i from S to T ,
   Return to step 1 ,

Else,

Output P,

End algorithm.

Example 1:

For graph shown in Fig.(1) draw the associated adjacency matrix , then find max flow.

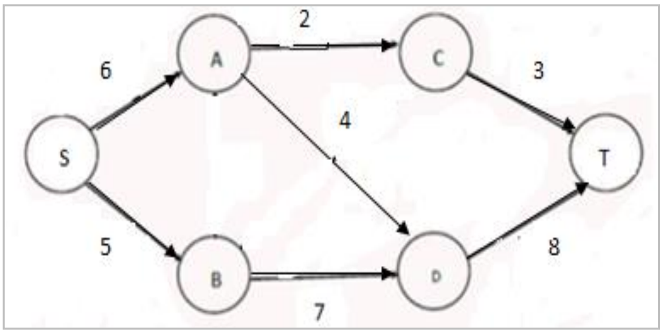

Figure 1:

The adjacency matrix will be:

$$\begin{bmatrix} S & A & B & C & D & T \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 7 & 0 & 0 \\ 0 & 0 & 3 & 0 & 8 & 0 \end{bmatrix}$$

Subtract 2 from P_1 (SABT)

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S	A	B	T
0	0	0	0
4	0	0	0
0	0	0	0
0	0	1	0

Subtract 4 from P_2 (SADT)

S	A	D	T
0	0	0	0
0	0	0	0
0	0	0	0
0	0	4	0

Subtract 4 from P_3(SCDT)

S	C	D	T
0	0	0	0
1	0	0	0
0	3	0	0
0	0	4	0

Then max flow = 2 +4 +4 =10.

Example 2:

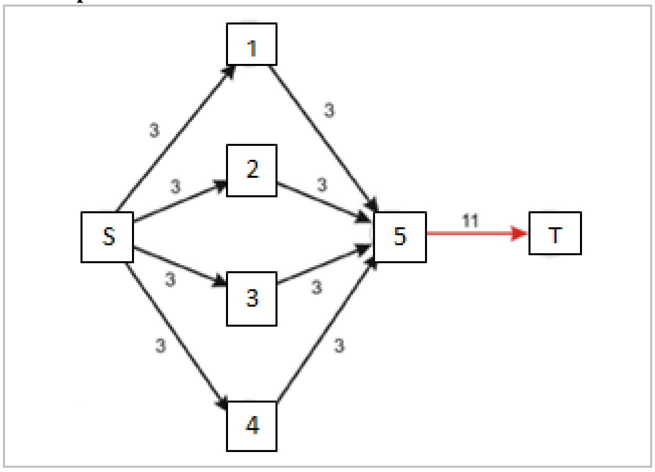


Figure 2:

The adjacency matrix for graph in Fig.(2) is:

S	1	2	3	4	5	T
0	0	0	0	0	0	0
3	0	0	0	0	0	0
3	0	0	0	0	0	0
3	0	0	0	0	0	0
3	0	0	0	0	0	0
0	3	3	3	3	0	0
0	0	0	0	0	11	0

Subtract 3 from P_1(S15T) ,

S	1	5	T
0	0	0	0
0	0	0	0
0	0	0	0
0	0	8	0

Subtract 3 from P_2(S25T)

S	2	5	T
0	0	0	0
0	0	0	0
0	0	0	0
0	0	5	0

Subtract 3 from p_3(S35T)

S	3	5	T
0	0	0	0
0	0	0	0
0	0	0	0
0	0	2	0

Subtract 2 from P_4(S45T)

S	4	5	T
0	0	0	0
1	0	0	0
0	1	0	0
0	0	2	0

Then max flow = 3 + 3+ 3+ 2 = 11.

Theorem1:

In flow problems, if there exist a cycle flow in a sink such that the addition of two flows equal to the third, then the adjacency matrix of path containing these nods have negative numbers.

Example 3:

The adjacency matrix for graph shown in Fig. (3) is:

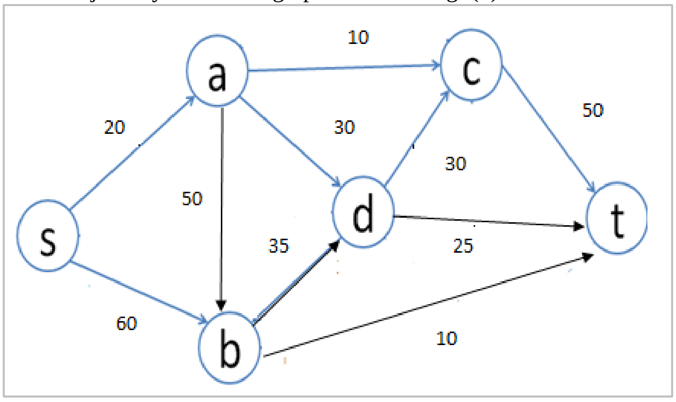


Figure 3:

S	a	b	d	c	t
0	0	0	0	0	0
20	0	0	0	0	0
60	50	0	0	0	0
0	30	35	0	0	0
0	10	0	30	0	0
0	0	10	25	50	0

Subtract 10 from path (sact)

S	a	c	t
0	0	0	0
10	0	0	0
0	0	0	0
0	0	40	0

Subtract 10 from (sbt)

S	b	t
0	0	0
50	0	0
0	0	0

Then subtract 10 from (sadt)

S	a	d	t
0	0	0	0
0	0	0	0
0	20	0	0
0	0	15	0

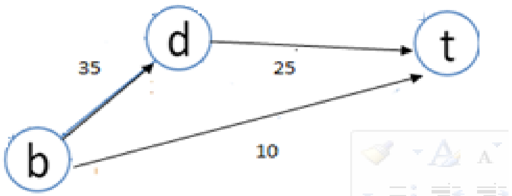
Then subtract 15 from (sbdt)

S	b	d	t
0	0	0	0
35	0	0	0
0	20	0	0
0	0	0	0

Finally, subtract 20 from (sbdct)

S	b	d	c	t
0	0	0	0	0
15	0	0	0	0
0	0	0	0	0
0	0	10	0	0
0	-20	-20	20	0

From the above theorem,
The flow graph contain a cycle such that:



bt+ dt = bd

Then there exist negative numbers (of red colour) in the adjacency matrix containing nods

b , d , t .

Example 4:

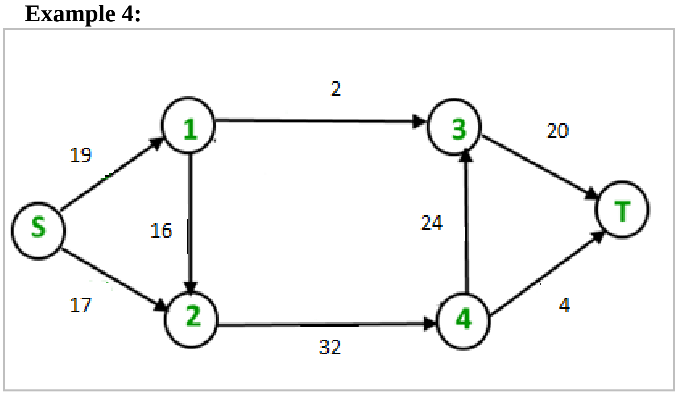


Figure 4:

The adjacency matrix is:

S	1	2	4	3	t
0	0	0	0	0	0
19	0	0	0	0	0
17	16	0	0	0	0
0	0	32	0	0	0
0	2	0	24	0	0
0	0	0	4	20	0

Subtract 2 from (s13T)

S	1	3	T
0	0	0	0
17	0	0	0
0	0	0	0
0	0	18	0

Then subtract 4 from (S24T)

S	2	4	T
0	0	0	0
13	0	0	0
0	28	0	0
0	0	0	0

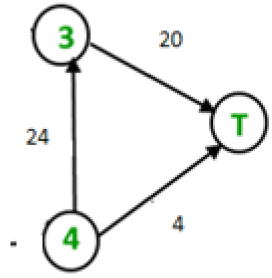
Subtract 13 from (S243T)

S	2	4	3	T
0	0	0	0	0
0	0	0	0	0
0	15	0	0	0
0	0	11	0	0
0	0	-13	5	0

Finally subtract 5 from (S1243T)

S	1	2	4	3	T
0	0	0	0	0	0
12	0	0	0	0	0
-5	11	0	0	0	0
0	0	10	0	0	0
0	-5	0	6	0	0
0	0	0	-5	0	0

There exist cycle such that $x_{4T} + X_{3T} = X_{43}$



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